Adjacency Eigenvalues for Underlying Split Multigraphs

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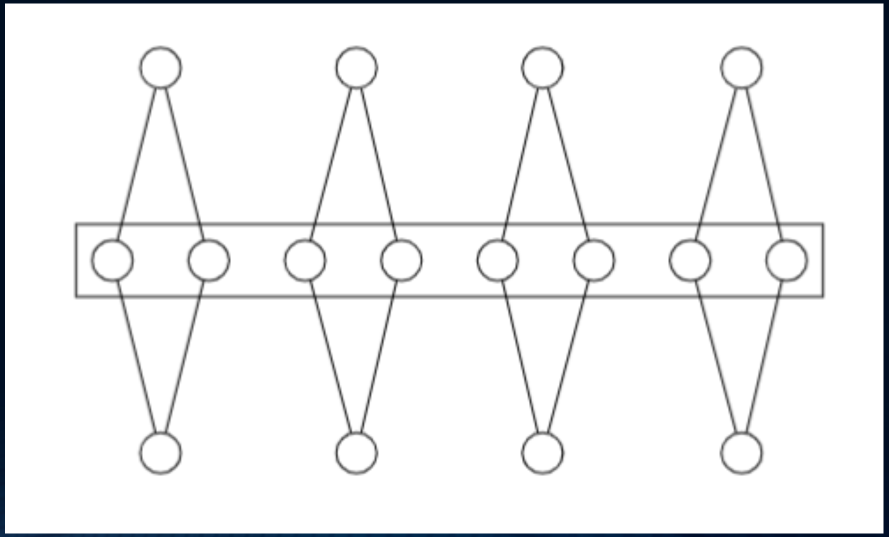
Background Information

$x\ - IPS(c, d)^{\mu}$

- *c* = number of cones
- *d* = degree of each cone
- μ = multiplicity of edges within clique
- x = number of cone nodes to which each clique node is adjacent

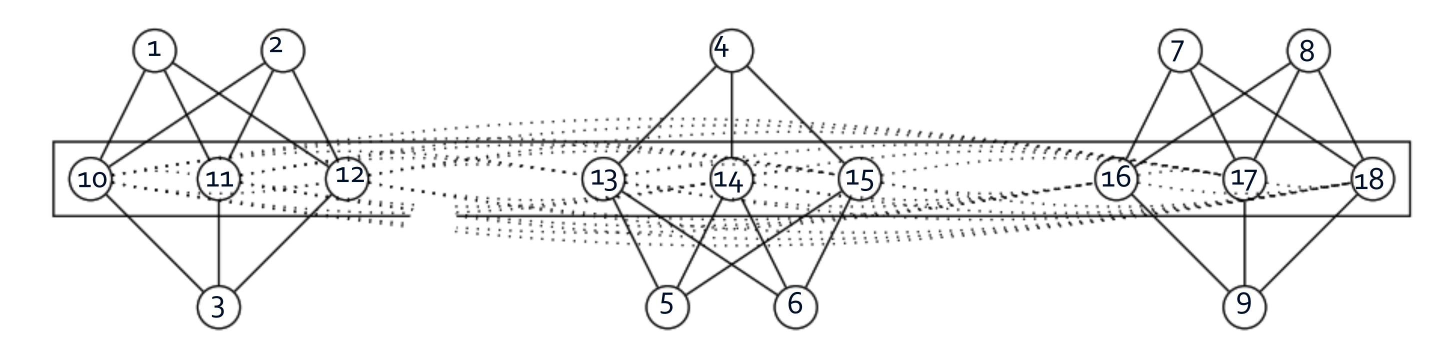
$2\ - IPS(8, 2)^{\mu}$



Research Question: If these multigraphs represent satellite and ground station communication, can we find a formula to best represent the number of triangles from the ground station to the satellite?

Method

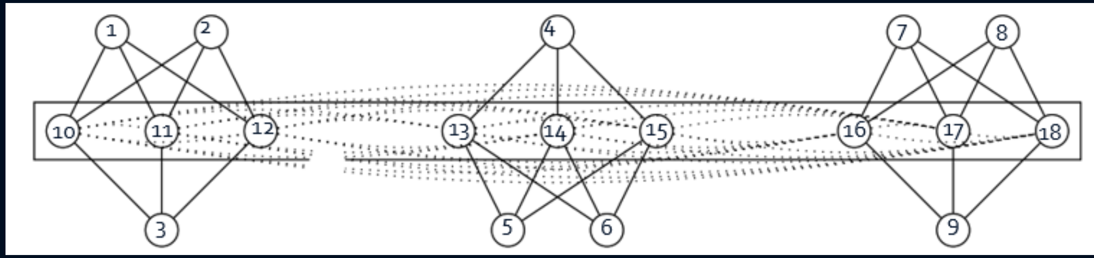
Adjacency Matrix



$3\ - IPS(9,3)^1$

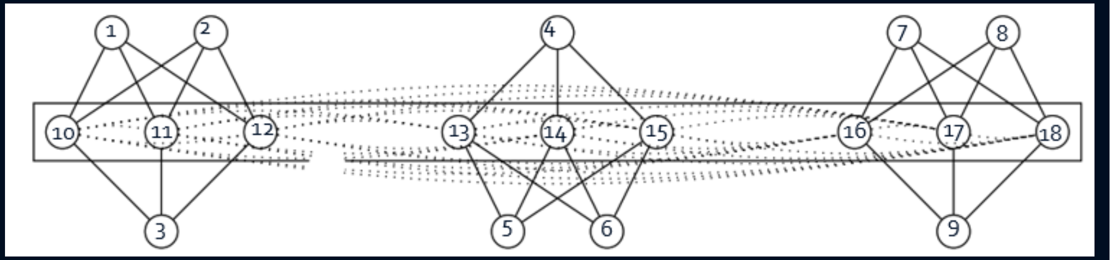
$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1\\
1&1&1&0&0&0&0&0&0&0&1&1&1&1&1&1&1&1\\
1&1&1&0&0&0&0&0&0&1&0&1&1&1&1&1&1&1\\
1&1&1&0&0&0&0&0&0&1&1&0&1&1&1&1&1&1\\
0&0&0&1&1&1&0&0&0&1&1&1&0&1&1&1&1&1\\
0&0&0&1&1&1&0&0&0&1&1&1&1&0&1&1&1&1\\
0&0&0&1&1&1&0&0&0&1&1&1&1&1&0&1&1&1\\
0&0&0&0&0&0&1&1&1&1&1&1&1&1&1&0&1&1\\
0&0&0&0&0&0&1&1&1&1&1&1&1&1&1&1&0&1\\
0&0&0&0&0&0&1&1&1&1&1&1&1&1&1&1&1&0
\end{pmatrix}$$

Adjacency Matrix



$$
\begin{pmatrix}
0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1\\
1&1&1&0&0&0&0&0&0&0&1&1&1&1&1&1&1&1\\
1&1&1&0&0&0&0&0&0&1&0&1&1&1&1&1&1&1\\
1&1&1&0&0&0&0&0&0&1&1&0&1&1&1&1&1&1\\
0&0&0&1&1&1&0&0&0&1&1&1&0&1&1&1&1&1\\
0&0&0&1&1&1&0&0&0&1&1&1&1&0&1&1&1&1\\
0&0&0&1&1&1&0&0&0&1&1&1&1&1&0&1&1&1\\
0&0&0&0&0&0&1&1&1&1&1&1&1&1&1&0&1&1\\
0&0&0&0&0&0&1&1&1&1&1&1&1&1&1&1&0&1\\
0&0&0&0&0&0&1&1&1&1&1&1&1&1&1&1&1&0
\end{pmatrix}
\longrightarrow
\begin{pmatrix}
0&1&1&1&1&1&1&1&1\\
1&0&1&1&1&1&1&1&1\\
1&1&0&1&1&1&1&1&1\\
1&1&1&0&1&1&1&1&1\\
1&1&1&1&0&1&1&1&1\\
1&1&1&1&1&0&1&1&1\\
1&1&1&1&1&1&0&1&1\\
1&1&1&1&1&1&1&0&1\\
1&1&1&1&1&1&1&1&0
\end{pmatrix}
$$

Characteristic Polynomial



- Submatrix:

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CharacteristicPolynomial[CP91, x]
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$8 + 63x + 216x^2 + 420x^3 + 504x^4 + 378x^5 + 168x^6 + 36x^7 - x^9$

168 / 2

84

- Whole Matrix:

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CharacteristicPolynomial[IPS931, x]
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$-729x^6 - 4860x^7 - 13473x^8 - 19502x^9 - 14427x^{10} - 2862x^{11} + 3360x^{12} + 2358x^{13} + 243x^{14} - 222x^{15} - 63x^{16} + x^{18}$

222 / 2

111

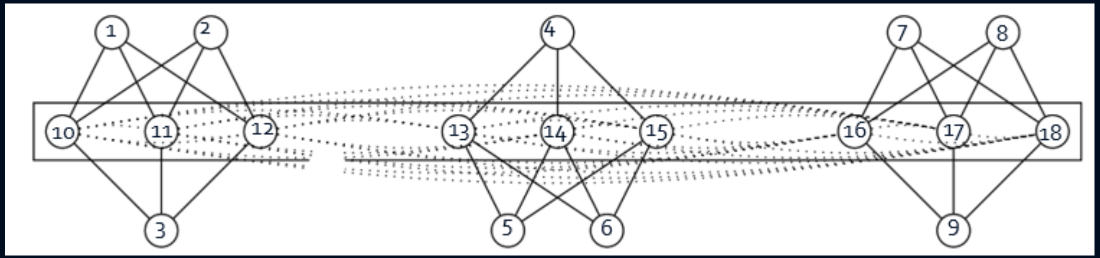
111 - 84

27

Conjecture

1. $Number\ of\ triangles\ can\ be\ found\ with\ the\ formula: \frac{(x-1)}{2} * (x^2 * c_o * \mu)$
2. $c_o(x-1) = number\ of\ eigenvalues\ equalling\ zero$
3. $c_o(x-1) = number\ of\ eigenvalues\ equalling\ -\mu$
4. $There\ are\ two\ eigenvalues\ that\ are\ the\ roots\ of\ \mathrm{x}^2 + \mu\mathrm{x} + -(x^2)$
5. $There\ is\ one\ eigenvalue\ that\ is\ the\ roots\ of\ \mathrm{x}^2 + (-\mu)(c-1)\mathrm{x} - x^2$

Triangles

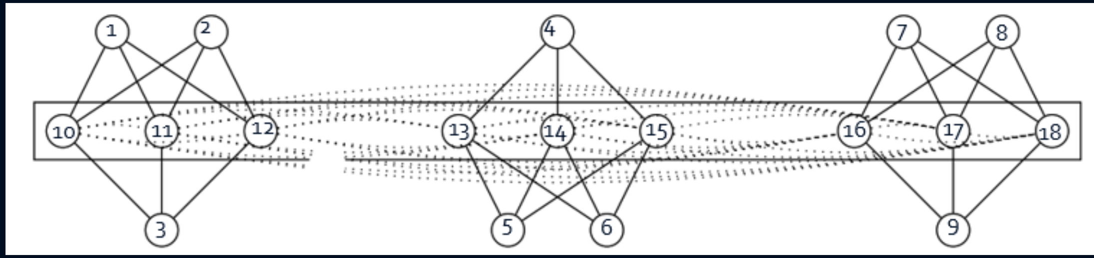


- Conjecture 1: $\frac{(x-1)}{2} * (x^2 * c_o * \mu)$

$$\frac{(3-1)}{2} * (3^2 * 3 * 1)$$
$$1 * (9 * 1)$$
$$\mathbf{27}$$

$$* \, Note: \; c = x * c_o$$
$$9 = 3 * 3$$

Eigenvalues

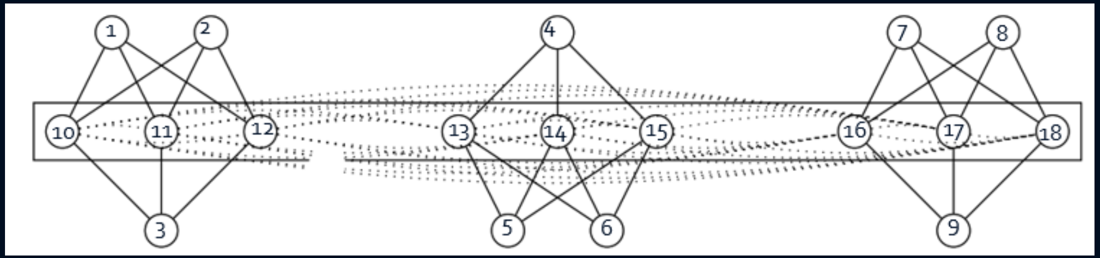


{9., -3.54138, -3.54138, 2.54138, 2.54138, -1., -1., -1., -1., -1., -1., -1., 0., 0., 0., 0., 0., 0.}

Eigenvalues = 0 have a multiplicity of 6
Eigenvalues = -1 have a multiplicity of 6

$$3 - IPS(9,3)^{1}$$
$$c_o = 3$$
$$x = 3$$

Conjecture 2 & 3: $c_o(x-1) = number\ of\ eigenvalues\ equalling\ zero$ & $equalling\ -\mu$



Eigenvalues

- Conjecture 4:
  There are two eigenvalues that are the roots of $\mathrm{x}^2 + \mu\mathrm{x} + -(x^2)$

{9., -3.54138, -3.54138, 2.54138, 2.54138, -1., -1., -1., -1., -1., -1., -1., 0., 0., 0., 0., 0., 0.}

$\mathrm{x}^2 + \mu\mathrm{x} + (-x^2)$

$1\mathrm{x}^2 + 1\mathrm{x} - 9$

Factored Polynomial Quotient: $(x^2 + x - 9)$

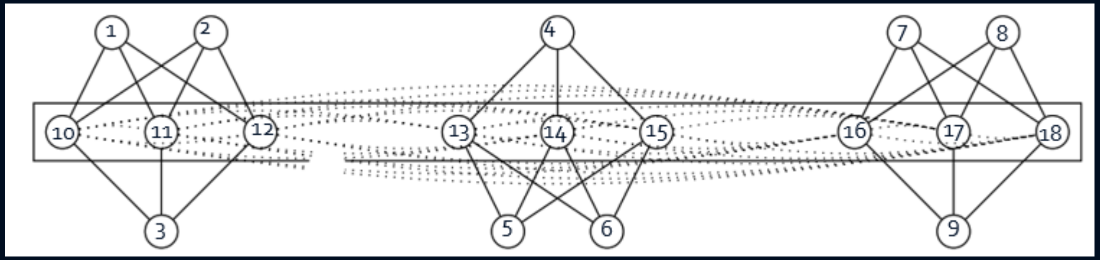
A = 1

B = 1

C = -9

$$\frac{\left(B \pm \sqrt{B^2 - 4AC}\right)}{2A}$$

-3.54138, 2.54138



Eigenvalues

- Conjecture 5: *There is one eigenvalue that is the roots of* $x^2 + (-\mu)(c-1)x - x^2$

{9., -3.54138, -3.54138, 2.54138, 2.54138, -1., -1., -1., -1., -1., -1., -1., 0., 0., 0., 0., 0., 0.}

$$x^2 + (-\mu)(c-1)x - x^2$$
$$1x^2 + (-1)(9-1)(1)x - 3^2$$
$$x^2 - 8x - 9$$

Irreducible: $(x^2 - 8x - 9)$

A = 1

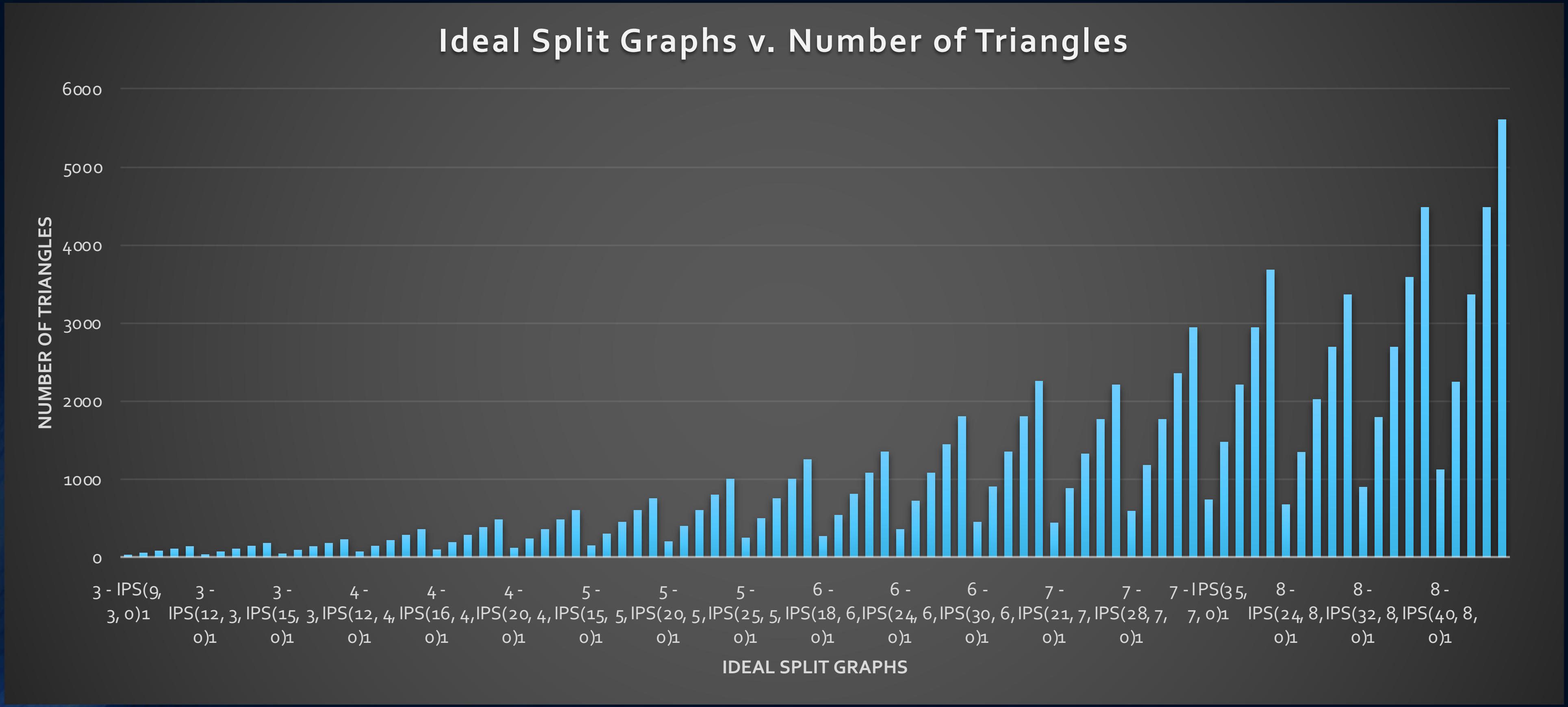
B = -8

C = -9

$$\frac{\left(B \pm \sqrt{B^2 - 4AC}\right)}{2A}$$

9, -1

Data



Ideal Split Graphs v. Number of Triangles
6000
5000
4000
3000
2000
1000
0
NUMBER OF TRIANGLES
3 - IPS(9, 3, 0)1
3 - IPS(12, 3, 0)1
3 - IPS(15, 3, 0)1
4 - IPS(12, 4, 0)1
4 - IPS(16, 4, 0)1
4 - IPS(20, 4, 0)1
5 - IPS(15, 5, 0)1
5 - IPS(20, 5, 0)1
5 - IPS(25, 5, 0)1
6 - IPS(18, 6, 0)1
6 - IPS(24, 6, 0)1
6 - IPS(30, 6, 0)1
7 - IPS(21, 7, 0)1
7 - IPS(28, 7, 0)1
7 - IPS(35, 7, 0)1
8 - IPS(24, 8, 0)1
8 - IPS(32, 8, 0)1
8 - IPS(40, 8, 0)1
IDEAL SPLIT GRAPHS

Thank you! Questions?